March 2009 - Solutions

1) How many different four-digit numbers can be formed by arranging the digits 2, 2, 8, and 9?

Solution:

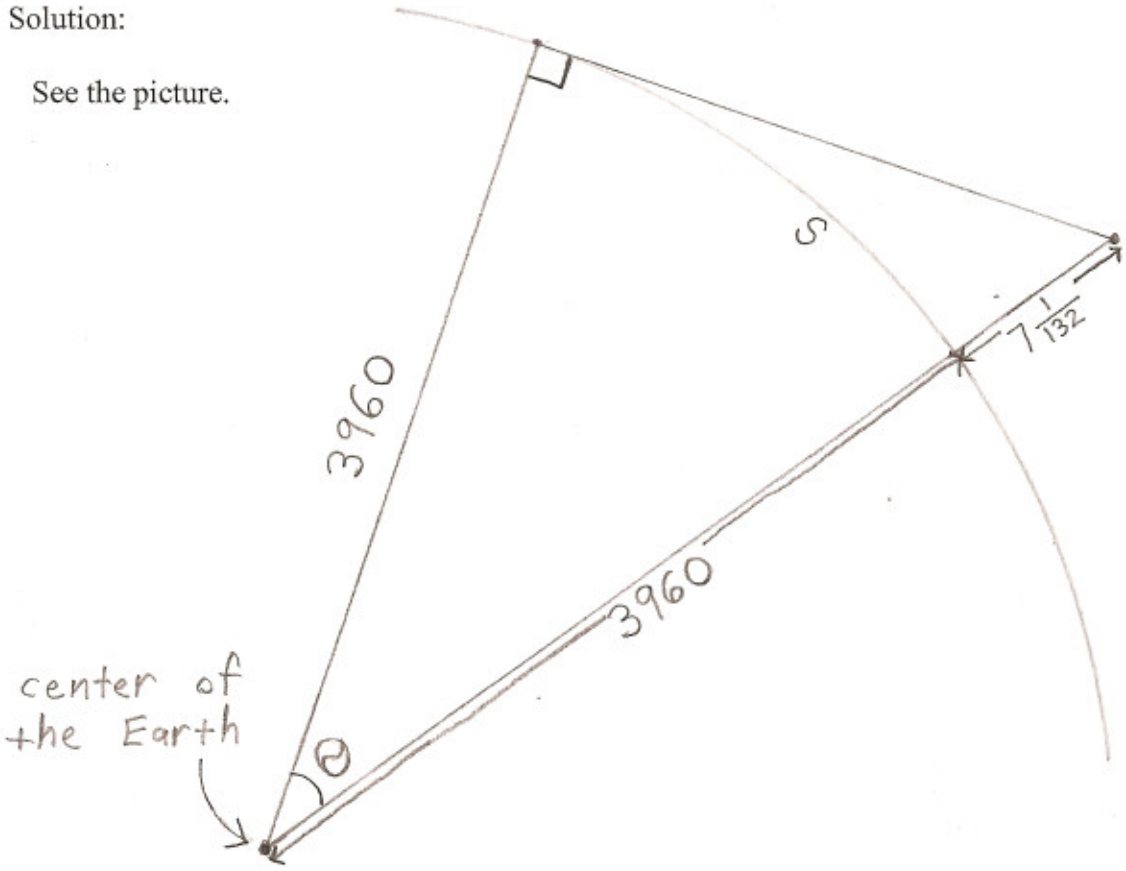
2289, 2298, 2829, 2892, 2928, 2982, 8229, 8292, 8922, 9228, 9282, 9822

There are 12 different four-digit numbers can be formed by arranging the digits 2, 2, 8, and 9.

2) Suppose you are flying in an airplane at an altitude of 37,000 feet. How far, along the Earth's surface, is your line-of-sight horizon from the point on the Earth's surface directly below you? In other words, how far can you see? Assume the Earth's surface is a perfect sphere with a radius of 3960 miles.

Solution:

See the picture.



$37{,}000 \text{ feet} = \frac{37{,}000}{5280} \text{ miles} = 7\frac{1}{132} \text{ miles.}$

$\cos\theta = \frac{3960}{3967\frac{1}{132}}$.

$\theta = \arccos\frac{3960}{3967\frac{1}{132}}$.

$s = 3960 \arccos\frac{3960}{3967\frac{1}{132}} \text{ miles}$

$\approx 235 \text{ miles}$