

## Course Outline

Course Title: Abstract Algebra With Introductory Number Theory

Common Course Title: MAS3301

Effective Term: Fall 2022 ( Aug 8, 2022 )

Credit Hours: 3 Units

Next Review : Aug 8, 2027

Contact Hour Breakdown: *(Per 16 week Term)*

Total: 48

Lecture:

Lab:

Clinic:

Other:

## Requirements

Pre-requisite(s) with minimum grade required

MAC2311 (C), AND MAD2104 (C), AND MAS 2103 (C)

## Course Description:

This course introduces students to the study of mathematical structures and their properties. Topics include introductory number theory, sets, operations, groups, rings, fields, and homomorphisms between these structures. Emphasis will be on theory and proof.

## Course Outline

### UNITS

#### **Unit 1 : Number Theory and Mathematical Proofs**

##### General Outcome

1.0 Prove and apply number theory-based theorems in the set of integers.

##### Specific Learning Outcomes

1.1 Apply the Division Algorithm in the set of integers.

1.2 Define the terms divisible, divisibility, factor, and greatest common divisor in the set of integers.

1.3 Prove and apply theorems of divisibility in the set of integers.

1.4 Define the term prime in the set of integers, and prove and apply theorems of primality in the set of integers.

1.5 Define the term congruence modulo  $n$ .

1.6 Prove and apply theorems of congruence modulo  $n$ .

1.7 Prove and apply theorems of  $\mathbb{Z}_p$ , with  $p$  prime.

1.8 Perform modular addition and multiplication.

#### **Unit 2 : The Algebra of Polynomials**

##### General Outcome

2.0 Identify the properties of and terminology associated with, polynomials, and prove and apply various theorems of polynomials.

#### Specific Learning Outcomes

- 2.1 Define the ring of polynomials over rings, integral domains, and fields.
- 2.2 Perform operations of polynomials over rings, integral domains, and fields.
- 2.3 Prove and apply the Division Algorithm and Factor Theorem for polynomials.
- 2.4 Define the term irreducible over an integral domain, and determine the irreducibility of a polynomial in an integral domain.
- 2.5 Prove and apply theorems of divisibility in  $F[x]$ , the ring of polynomials over a field.
- 2.6 Prove and apply theorems of the greatest common divisor in  $F[x]$ .
- 2.7 Define the term irreducible in  $F[x]$ , and prove and apply theorems of irreducibility in  $F[x]$ .
- 2.8 Define the term root in  $F[x]$ , and prove and apply theorems of roots, including the Remainder Theorems and Factor Theorem.

### **Unit 3 : Groups**

#### General Outcome

- 3.0 Identify the properties of and terminology associated with groups, and prove and apply various theorems of groups.

#### Specific Learning Outcomes

- 3.1 Define the term group, and determine if a given system (with one given operation) is a group.
- 3.2 Define the term order as it is applied to a group and to group elements, and determine the order of a finite group.
- 3.3 Define the term Abelian, and determine if a group is Abelian.
- 3.4 Identify and manipulate special examples of groups, including permutation groups, symmetry groups, and cyclic groups.
- 3.5 Prove and apply properties and theorems of groups.
- 3.6 Define the term subgroup, and prove and apply theorems of subgroups.
- 3.7 Define the term generator, and prove and apply theorems of subgroups generated by a subset of a group.
- 3.8 Define the terms cyclic group and cyclic subgroup, and prove and apply theorems of cyclic groups and cyclic subgroups.
- 3.9 Determine the center of a group.
- 3.10 Determine the centralizer of a given group element.
- 3.11 Define the terms homomorphism and isomorphism, and determine if a mapping between groups is a homomorphism and/or an isomorphism.

### **Unit 4 : Rings**

#### General Outcome

- 4.0 Identify the properties of and terminology associated with rings, and prove and apply various theorems of rings.

#### Specific Learning Outcomes

- 4.1 Define the term ring, and determine if a given system (with two operations) is a ring.
- 4.2 Prove and apply properties and theorems of rings.
- 4.3 Define the term subring, and determine if a given ring is a subring of another.
- 4.4 Prove and apply theorems of subrings.
- 4.5 Define the term integral domain, and determine if a given ring is an integral domain.

4.6 Prove and apply theorems of integral domains.

4.7 Define the term isomorphism, and determine if a function mapping one ring to another is an isomorphism.

4.8 Define the term homomorphism, and determine if a function mapping one ring to another is a homomorphism.

## **Unit 5 : Fields**

### General Outcome

5.0 Identify the properties of and terminology associated with fields, and prove and apply various theorems of fields.

### Specific Learning Outcomes

5.1 Define the term field, and determine if a given system (with two operations) is a field.

5.2 Prove and apply properties and theorems of fields.

5.3 Define the term subfield, and determine if a given field is a subfield of another.

5.4 Prove and apply theorems of subfields.