BROWARD COMMUNITY COLLEGE
COURSE OUTLINE

LAST REVIEW: 2008-09 *(i.e. 2003-2004)* **NEXT REVIEW:** 2013-14 *(i.e. 2008-2009)* **STATUS:** A *(A, I, D)*

COURSE TITLE: ABSTRACT ALGEBRA with INTRODUCTORY NUMBER THEORY

COMMON COURSE NUMBER: MAS 4300

CREDIT HOURS: 3

CONTACT HOUR BREAKDOWN *(per 16 week term)*

CLOCK HOURS: *(Voc. Course ONLY)*

Lecture: 48 Lab:

Clinic: Other:

PREREQISITE(S): MAC 2311 and MAD 2104

COREQUISITE(S): NONE

PRE/COREQUISITE(S): NONE

COURSE DESCRIPTION: *(750 characters, maximum)*

A course for math and math education majors. Abstract algebra is designed for the student experienced with using mathematical calculations to solve problems, and who now wishes to analyze the underlying justifications for these calculations' legitimacy. In MAS 4300 the student will discover properties shared by seemingly disparate mathematical structures called groups, rings, and fields, by *abstracting* their common underlying features, and creating proofs based upon these commonalities. Number theory topics that are foundational to this course will be studied as well.

UNIT TITLES

1. **Number Theory**
2. **Arithmetic in F[x]**
3. **Groups**
4. **Rings**
5. **Fields**

ASSESSMENT:

Please provide a brief description (250 characters maximum) that details how students will be assessed on the course outcomes.

Written Quizzes/Examinations.
Cumulative Final Examination.

****** Complete the following only if course is seeking general education status ******

GENERAL EDUCATION Competencies and Skills *:

Please highlight in green font all Competencies/Skills from the list below that apply to this course. In the box to the right of the Competency/Skill, enter all specific learning outcome numbers (i.e. 1.1, 2.7, 5.12) that apply.

1. Read with critical comprehension	
2. Speak and listen effectively	
3. Write clearly and coherently	1.1; 1.3; 1.4; 1.5; 1.6; 1.7; 1.8; 1.10, 1.11, 1.14 2.3; 2.5; 2.6; 2.7; 2.8; 2.9; 2.10; 2.13; 2.15 3.4; 3.5; 3.6; 3.7; 3.8; 3.9; 3.10; 3.11; 3.12; 3.13; 3.14; 3.15; 3.16 4.2; 4.4; 4.6; 4.8; 4.10; 4.12; 4.13; 4.15 5.1; 5.3
4. Think creatively, logically, critically, and reflectively (analyze, synthesize, apply, and evaluate)	1.1; 1.3; 1.4; 1.5; 1.6; 1.7; 1.8; 1.10, 1.11, 1.14 2.3; 2.5; 2.6; 2.7; 2.8; 2.9; 2.10; 2.13; 2.15 3.3; 3.4; 3.5; 3.6; 3.7; 3.8; 3.9; 3.10; 3.11; 3.12; 3.13; 3.14; 3.15; 3.16 4.2; 4.3; 4.4; 4.5; 4.6; 4.7; 4.8; 4.9; 4.10; 4.11; 4.12; 4.13; 4.15 5.2; 5.4
5. **Demonstrate and apply literacy in its various forms:** *(highlight in green ALL that apply)* (1. technological, 2. informational, 3. mathematical, 4. scientific, 5. cultural, 6. historical, 7. aesthetic and/or 8. environmental)	1.2; 1.6; 1.9; 1.12, 1.13 2.1; 2.2; 2.4; 2.7; 2.8; 2.11; 2.12; 2.14 3.1; 3.2; 3.5; 3.6; 3.7; 3.8; 3.11; 3.13; 3.14 4.1; 4.3; 4.5; 4.7; 4.9; 4.11; 4.14 5.2; 5.4
6. Apply problem solving techniques to real-world experiences	1.13
7. Apply methods of scientific inquiry	
8. Demonstrate an understanding of the physical and biological environment and how it is impacted by human beings	
9. Demonstrate an understanding of and appreciation for human diversities and commonalities	
10. Collaborate with others to achieve common goals.	
11. Research, synthesize and produce original work	
12. Practice ethical behavior	
13. Demonstrate self-direction and self motivation	
14. Assume responsibility for and understand the impact of personal behaviors on self and society	
15. Contribute to the welfare of the community	

* *General Education Competencies and Skills endorsed by '05-'06 General Education Task Force*

Common Course Number: MAS 4300

Unit 1 Number Theory

General Outcome:

1.0 The student shall be able to prove and apply number theory-based theorems of prime numbers in $\mathbb{Z}$ and congruence classes in $\mathbb{Z}_n$.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

1.1 Prove and apply the Division Algorithm in $\mathbb{Z}$.

1.2 Define the terms "divisible," "divisibility," "factor" and "greatest common divisor" in $\mathbb{Z}$.

1.3 Prove and apply theorems of divisibility in $\mathbb{Z}$.

1.4 Prove and apply theorems of the greatest common divisor in $\mathbb{Z}$.

1.5 Prove and apply theorems of the Euclidean Algorithm.

1.6 Define the term "prime" in $\mathbb{Z}$, and prove and apply theorems of primality in $\mathbb{Z}$.

1.7 Prove and apply the Fundamental Theorem of Arithmetic.

1.8 Prove and apply primality tests, like the Prime Number Theorem. **(OPTIONAL)**

1.9 Define the terms "congruence *modulo* n" and "congruence class" in $\mathbb{Z}_n$.

1.10 Prove and apply theorems of congruence *modulo* n and of congruence classes in $\mathbb{Z}_n$.

1.11 Prove and apply theorems of $\mathbb{Z}_p$, with p prime.

1.12 Perform modular addition and multiplication.

1.13 Define and apply the properties of the system of Public Key Cryptography, including the RSA system. **(OPTIONAL)**

1.14 Prove and apply the Chinese Remainder Theorem. **(OPTIONAL)**

Common Course Number: MAS 4300

Unit 2 Arithmetic in F[x]

General Outcome:

2.0 The student shall be able to prove and apply number theory-based theorems of irreducibles in F[x] and congruence classes in F[x]/(p(x)).

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

2.1 Define the term "F[x]."

2.2 Perform addition and multiplication in F[x].

2.3 Prove and apply the Division Algorithm in F[x].

2.4 Define the terms "divisible," "divisibility," "factor" and "greatest common divisor" in F[x].

2.5 Prove and apply theorems of divisibility in F[x].

2.6 Prove and apply theorems of the greatest common divisor in F[x].

2.7 Define the term "irreducible" in F[x], and prove and apply theorems of irreducibility in F[x].

2.8 Define the term "root" in F[x], and prove and apply theorems of roots, including the Remainder Theorem and the Factor Theorem.

2.9 Prove and apply theorems specific to $\mathbb{Z}$[x], $\mathbb{Q}$[x], $\mathbb{R}$[x], and $\mathbb{C}$[x].

2.10 Prove and apply the Rational Root Theorem.

2.11 Apply the Fundamental Theorem of Algebra (without proof) to prove theorems in $\mathbb{R}$[x] and $\mathbb{C}$[x]. **(OPTIONAL)**

2.12 Define the terms "congruence *modulo* p(x)" and "congruence class" in F[x]/(p(x)).

2.13 Prove and apply theorems of congruence *modulo* p(x) and of F[x]/(p(x)).

2.14 Perform addition and multiplication in F[x]/(p(x)).

2.15 Prove and apply theorems of F[x]/(p(x)), with p(x) irreducible.

Common Course Number: MAS 4300

Unit 3 Groups

General Outcome:

3.0 The student shall be able to identify the properties of, and terminology associated with, algebraic groups, and prove and apply various theorems of groups.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

3.1 Define the term "group," and determine if a given system (with one given operation) is a group.

3.2 Define the term "abelian," and determine if a group is abelian.

3.3 Identify and manipulate special examples of groups, including permutation groups, symmetry groups, and cyclic groups.

3.4 Prove and apply properties and theorems of groups.

3.5 Define the term "subgroup," and prove and apply theorems of subgroups.

3.6 Define the terms "cyclic group" and "cyclic subgroup," and prove and apply theorems of cyclic groups and cyclic subgroups.

3.7 Define the term "generator," and prove and apply theorems of subgroups generated by a subset of a group.

3.8 Define the terms "homomorphism" and "isomorphism," and determine if a mapping between groups is a homomorphism and/or isomorphism.

3.9 Prove and apply theorems of homomorphisms and isomorphisms.

3.10 Prove and apply Cayley's Theorem.

3.11 Define the terms "coset" and "index," and prove and apply theorems of cosets and indices of subgroups of a group.

3.12 Prove and apply Lagrange's Theorem.

3.13 Define the term "normal subgroup," and prove and apply theorems of normal subgroups.

3.14 Define the term "quotient group," and prove and apply theorems of quotient groups.

3.15 Prove and apply the Isomorphism Theorems for Groups. **(OPTIONAL)**

3.16 Prove and apply the Sylow Theorems. **(OPTIONAL)**

Common Course Number: MAS 4300

Unit 4 Rings

General Outcome:

4.0 The student shall be able to identify the properties of, and terminology associated with, algebraic rings, and prove and apply various theorems of rings.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

4.1 Define the term "ring," and determine if a given system (with two operations) is a ring.

4.2 Prove and apply properties and theorems of rings.

4.3 Define the term "subring," and determine if a given ring is a subring of another.

4.4 Prove and apply theorems of subrings.

4.5 Define the term "integral domain," and determine if a given ring is an integral domain.

4.6 Prove and apply theorems of integral domains.

4.7 Define the term "isomorphism," and determine if a function mapping one ring to another is an isomorphism.

4.8 Prove and apply theorems of isomorphisms.

4.9 Define the term "homomorphism," and determine if a function mapping one ring to another is an homomorphism.

4.10 Prove and apply theorems of homomorphisms.

4.11 Define the term "ideal," and determine if a subring of a ring is an ideal.

4.12 Prove and apply theorems of ideals.

4.13 Prove and apply theorems of congruence of elements of a ring, R, *modulo* I, an ideal in R.

4.14 Define the terms "quotient ring," and create a quotient ring R/I by factoring the ideal, I, out of the ring, R.

4.15 Prove and apply the Isomorphism Theorems. **(OPTIONAL)**

Common Course Number: MAS 4300

Unit 5 Fields

General Outcome:

5.0 The student shall be able to identify the properties of, and terminology associated with, algebraic fields, and prove and apply various theorems of fields.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

5.1 Define the term "field," and determine if a given system (with two operations) is a field.

5.2 Prove and apply properties and theorems of fields.

5.3 Define the term "subfield," and determine if a given field is a subfield of another.

5.4 Prove and apply theorems of subfields.