BROWARD COMMUNITY COLLEGE
COURSE OUTLINE

LAST REVIEW: 2008-09 *(i.e. 2003-2004)* **NEXT REVIEW:** 2013-14 *(i.e. 2008-2009)* **STATUS:** A *(A, I, D)*

COURSE TITLE: MODERN GEOMETRIES

COMMON COURSE NUMBER: MTG 3212

CREDIT HOURS: 3

CONTACT HOUR BREAKDOWN *(per 16 week term)*

CLOCK HOURS: *(Voc. Course ONLY)*

Lecture: **48** Lab:

Clinic: Other:

PREREQISITE(S): MAD 2104 or permission of the Mathematics Department

COREQUISITE(S): None

PRE/COREQUISITE(S): None

COURSE DESCRIPTION: *(750 characters, maximum)*

A course for math and math education majors. Geometry is a major foundation of our mathematical understanding of the world, and this course will explore both its breadth and depth. This course rigorously examines the axioms and theorems of both Euclidean geometry and the non-Euclidean geometries. The coordinate and translational geometries will be treated as well. This course is highly theoretical and proof-intensive. Thus some background with constructing direct proofs and proofs by contradiction is a necessary prerequisite to enrolling in this course.

UNIT TITLES

1. **Euclidean Geometry: Points, Lines, Angles, & Planes**
2. **Euclidean Geometry: Triangles**
3. **Euclidean Geometry: Curves & Polygons**
4. **Euclidean Geometry: Circles**
5. **Euclidean Geometry: Solids**
6. **Euclidean Geometry: Higher-Order Dimensions (OPTIONAL)**
7. **Euclidean Geometry: Constructions Using Compass & Straightedge**
8. **Coordinate Geometry**
9. **Symmetry, Tessellations, & Translations**
10. **Non-Euclidean Geometries: Elliptical, Hyperbolic, & Fractal Geometries**
11. **Non-Euclidean Geometries: Affine & Projective Geometries (OPTIONAL)**
12. **Topology (OPTIONAL)**

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UNITS

Unit 1 Euclidean Geometry: Points, Lines, Angles, & Planes

General Outcome:

1.0 The student shall be able to define terms and prove theorems associated with points, lines, angles, & planes, and prove theorems regarding angles formed when a transversal intersects parallel lines.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

1.1 Define the terms “point,” “line,” and “plane.”

1.2 State basic Euclidean postulates for points & lines, and prove theorems using them.

1.3 Define the terms “(line) segment,” “ray,” & “half-line,” and prove theorems using them.

1.4 Define the terms “parallel,” “perpendicular,” & “skew,” and determine if a pair of lines is parallel, perpendicular, skew, or neither.

1.5 Prove theorems regarding parallel and perpendicular lines.

1.6 Define the terms “angle” and “vertex.”

1.7 Define the terms “acute angle,” “right angle,” “obtuse angle,” “straight angle,” & “reflex angle,” and determine whether an angle is acute, right, obtuse, straight or reflex.

1.8 Define the terms “adjacent,” “congruent,” “complementary,” & “supplementary,” and determine if a pair of angles is adjacent, congruent, complementary, and/or supplementary.

1.9 Define the term “vertical angle.”

1.10 Prove theorems regarding and using angles, including vertical angles.

1.11 Define the terms “transversal (of parallel lines),” “alternate exterior angle,” “alternate interior angle,” “same-side interior angle,” “same-side exterior angle,” and “corresponding angle.”

1.12 Determine relationships among angles formed when a transversal intersects parallel lines.

1.13 State Euclidean’s Parallel Postulate.

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Unit 2 Euclidean Geometry: Triangles

General Outcome:

2.0 The student shall be able to define terms and prove theorems associated with triangles.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

2.1 Define the terms “triangle,” “acute triangle,” “right triangle,” “obtuse triangle,” “equilateral triangle,” “isosceles triangle,” and “scalene triangle.”

2.2 Classify triangles based on angle measurements.

2.3 Classify triangles based on the number of congruent sides.

2.4 Define the terms “area” & “perimeter,” and find the area and perimeter of triangles.

2.5 Prove theorems of triangle congruence (e.g. the Side-Angle-Side Congruence Theorem), and determine if given triangles are congruent.

2.6 Define the terms “interior angle” & “exterior angle,” and prove theorems regarding and using the interior and exterior angles of triangles.

2.7 Prove that the sum of the interior angles of any triangle is 180^{o}.

2.8 Explain the triangle inequality, and prove theorems regarding and using the triangle inequality.

2.9 Prove and apply theorems regarding right triangles, including the Pythagorean Theorem.

2.10 Define the term “similarity,” and prove and apply the properties of similar figures.

2.11 Prove theorems of triangle similarity (e.g. the Angle-Angle Similarity Theorem), and determine if given triangles are similar.

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Unit 3 Euclidean Geometry: Curves & Polygons

General Outcome:

3.0 The student shall be able to define terms and prove theorems associated with curves & polygons.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

3.1 Define the terms “curve,” “simple curve,” “closed curve,” and “convex curve.”

3.2 Determine if a curve is simple, closed, and/or convex, and create curves including or excluding any combination of these characteristics.

3.3 Define the terms “polygon” and “regular (polygon).”

3.4 Determine the sum of measures of the interior angles of any polygon.

3.5 Determine the measure of each interior angle of any regular polygon.

3.6 Define the term “apothem,” and determine the area and perimeter of appropriate polygons.

3.7 Define the terms “quadrilateral,” “parallelogram,” “rectangle,” “rhombus,” “square,” “kite,” “trapezoid,” and “diagonal.”

3.8 Prove that a given quadrilateral is or is not a parallelogram, kite, or trapezoid.

3.9 Prove that a given parallelogram is or is not a rectangle, rhombus, or square.

3.10 Prove theorems regarding and using the sides, interior angles, diagonals, perimeters & areas of parallelograms, rectangles, rhombi, squares, kites, & trapezoids.

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Unit 4 Euclidean Geometry: Circles

General Outcome:

4.0 The student shall be able to define terms and prove theorems associated with circles.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

4.1 Define the terms “circle,” “center (of a circle),” “radius,” “diameter,” and “circumference.”

4.2 Determine the circumference and area of circles, and prove theorems regarding and using them.

4.3 Define the terms “central angle,” “inscribed angle,” “arc,” “minor arc,” and “major arc.”

4.4 Determine the measure of central angles, inscribed angles, and arcs.

4.5 Prove theorems regarding and using central angles, inscribed angles, and arcs.

4.6 Define the term “sector,” and prove theorems regarding and using it.

4.7 Determine the area of sectors.

4.8 Define the terms “chord” & “secant (line),” and prove theorems regarding and using them.

4.9 Determine the measure of the angles and segments formed by intersecting chords and intersecting secants.

4.10 Define the terms “tangent (line)” & “point of tangency,” and prove theorems regarding and using them.

4.11 Determine, and prove theorems regarding and using, the measure of angles formed by an intersecting tangent & chord, an intersecting tangent & secant, and intersecting tangents.

4.12 Determine the measure of segments formed by an intersecting tangent & secant and intersecting tangents.

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Unit 5 Euclidean Geometry: Solids

General Outcome:

5.0 The student shall be able to define terms and prove theorems associated with solids.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

5.1 Define the terms “polyhedron,” “face,” “edge,” & “vertex,” and prove theorems regarding and using them.

5.2 Define the term “platonic solid (or regular polyhedron),” and identify the five platonic solids.

5.3 Explain and apply Euler’s formula.

5.4 Prove theorems about the platonic solids. (OPTIONAL)

5.5 Define the term “duality (of solids),” and construct the dual of any solid. (OPTIONAL)

5.6 Define the terms “prism,” “cylinder,” “base,” “lateral face,” & “height,” and prove theorems regarding and using them.

5.7 Define and distinguish between the following pairs of terms: “right prism” & “oblique prism”; “right circular cylinder” & “oblique circular cylinder.”

5.8 Define the terms “surface area” & “volume,” and determine the surface area and volume of prisms & cylinders.

5.9 Define the terms “pyramid,” “cone,” “lateral face,” “apex,” & “slant height,” and prove theorems regarding & using them.

5.10 Define and distinguish between the following pairs of terms: “right regular pyramid” & “oblique regular pyramid” ; “right cone” & “oblique cone.”

5.11 Determine the surface area and volume of pyramids and cones.

5.12 Define the term “net.”

5.13 Determine the polyhedron built from a net, and vice versa.

5.14 Determine the Mercator projections of polyhedra. (OPTIONAL)

5.15 Define the terms “sphere” and “center (of a sphere),” and prove theorems regarding and using them.

5.16 Determine the surface area and volume of spheres.

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Unit 6 Euclidean Geometry: Higher-Order Dimensions (OPTIONAL)

General Outcome:

6.0 The student shall be able to (1) describe the concept of dimensionality; (2) describe the defining characteristics of any dimension; (3) describe the properties of shapes with dimension > 3.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

6.1 Define the terms “dimension” and “dimensionality.”

6.2 Determine defining characteristics of the 0^{th}, 1^{st}, 2^{nd}, 3^{rd}, and 4^{th} dimensions.

6.3 Explain how the n^{th} dimension is extended from the $(n–1)^{st}$ dimension.

6.4 Define the terms “n-cube” and “n-sphere (or n-ball),” and describe the properties of each.

6.5 Prove theorems regarding solids with dimension > 3.

6.6 Determine defining characteristics of non-whole number dimensions.

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Unit 7 Euclidean Geometry: Constructions Using Compass & Straightedge

General Outcome:

7.0 The student shall be able to use a compass and straightedge to do line, angle, polygon, and circle constructions.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

7.1 Use compass and straightedge to copy line segments and angles.

7.2 Use compass and straightedge to construct a perpendicular to a line through a given point on the line.

7.3 Use compass and straightedge to construct a perpendicular to a line from a point not on the line.

7.4 Define the terms “bisection” and “perpendicular bisector.”

7.5 Use compass and straightedge to construct a perpendicular bisector of a segment.

7.6 Use compass and straightedge to bisect angles.

7.7 Use compass and straightedge to construct angles of any degree measure.

7.8 Use compass and straightedge to construct triangles of any type.

7.9 Define the terms “circumscribe (about a polygon),” “inscribe (within a polygon),” “centroid,” and “orthocenter.”

7.10 Use compass and straightedge to construct circumscribed circles of regular polygons.

7.11 Use compass and straightedge to construct inscribed circles of regular polygons.

7.12 Use compass and straightedge to construct centroids of regular polygons.

7.13 Use compass and straightedge to construct orthocenters of regular polygons.

7.14 Use compass and straightedge to construct the tangent to a circle at a point on a circle.

7.15 Use compass and straightedge to construct the tangent to a circle from a point outside the circle.

7.16 Use compass and straightedge to construct a common internal tangent to two circles.

7.17 Use compass and straightedge to construct a common external tangent to two circles.

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Unit 8 Coordinate Geometry

General Outcome:

8.0 The student shall be able to define terms and prove theorems associated with coordinate geometry.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

8.1 Define the terms “coordinate plane,” “x-axis,” “y-axis,” “origin,” “quadrant,” “ordered pair,” “x-coordinate,” and “y-coordinate.”

8.2 Derive the distance formula, and use it to find the distance between two points on a coordinate plane.

8.3 Derive the midpoint formula, and use it to find the coordinates of the midpoint between two points on a coordinate plane.

8.4 Prove theorems regarding and using distance and midpoint on a coordinate plane.

8.5 Define the term “slope (of a line).”

8.6 State the slope formula, and use it to find the slope of a line or segment on a coordinate plane.

8.7 Prove the slope relationships of parallel or perpendicular lines.

8.8 Prove theorems regarding and using the slope of lines or segments on a coordinate plane.

8.9 Construct polygons and circles using coordinate geometry.

8.10 Prove theorems regarding polygons and circles using coordinate geometry.

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Unit 9 Symmetry, Tessellations, & Translations

General Outcome:

9.0 The student shall be able to (1) describe all symmetries of a curve; (2) create tessellations; and (3) describe & perform translations on a curve.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

9.1 Define the terms “reflection symmetry,” “axis (or line) of symmetry,” “rotation symmetry,” and “center of rotation symmetry.”

9.2 Determine which symmetries, if any, are exhibited by a curve.

9.3 Determine all axes of symmetry for a curve.

9.4 Define the terms “tessellation (or tiling)” and “regular tessellation.”

9.5 Create tessellations of a plane.

9.6 Create tessellations of space. (OPTIONAL)

9.7 Define the terms “transformation,” “translation,” “rotation,” “reflection,” “pre-image,” and “image.”

9.8 Determine the specific transformational relationship between a pre-image and its image.

9.9 Perform any combination of transformations on a curve.

9.10 Define the term “isometry.” (OPTIONAL)

9.11 Prove theorems relating congruence and isometries. (OPTIONAL)

9.12 Define the terms “size transformation” “center (of a size transformation),” and “scale factor.” (OPTIONAL)

9.13 Describe and prove properties of size transformations of curves. (OPTIONAL)

9.14 Perform size transformations (with a given center and scale factor) on a curve. (OPTIONAL)

9.15 Define the term “similitude.” (OPTIONAL)

9.16 Prove theorems relating similarity and similitudes. (OPTIONAL)

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Unit 10 Non-Euclidean Geometry: Elliptical, Hyperbolic, & Fractal Geometries

General Outcome:

10.0 The student shall be able to describe and construct representations of the properties of the Elliptical and Hyperbolic geometries.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

10.1 Define the term "absolute geometry."

10.2 Construct equivalent formulations of Euclidean's Parallel Postulate, including Playfair's Axiom and Proclus' Axiom.

10.3 Explain how replacing the Parallel Postulate with an equivalent formulation yields the elliptical and hyperbolic geometries.

10.4 Define "line" within the meaning of the elliptical and hyperbolic geometries.

10.5 Describe the properties of parallelism within the meaning of the elliptical and hyperbolic geometries.

10.6 Describe the properties of triangles within the meaning of the elliptical and hyperbolic geometries.

10.7 Describe properties of polygons within the meaning of the elliptical and hyperbolic geometries. (OPTIONAL)

10.8 Create spherical models demonstrating properties of the elliptical geometry.

10.9 Create Poincaré, Klein, and/or pseudosphere models demonstrating properties of the hyperbolic geometry.

10.10 Define the term "fractal," and describe the properties of fractals.

10.11 Create fractals, including the Cantor Set, the Koch snowflake, and Sierpinski Triangle.

10.12 Prove theorems regarding fractals and fractal dimensions. (OPTIONAL)

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Unit 11 Non-Euclidean Geometry: Affine & Projective Geometries (OPTIONAL)

General Outcome:

11.0 The student shall be able to (1) describe the properties of the affine geometry; (2) describe the properties of the projective geometry; and (3) describe the properties of fractals and construct them.

Specific Measurable Learning Outcomes:
Upon successful completion of this unit, the student shall be able to:

11.1 Describe the defining characteristics of the affine geometry.

11.2 State basic postulates of the points and lines of an affine geometry.

11.3 State the Parallel Postulate for Affine Geometry.

11.4 Prove theorems regarding affine geometry.

11.5 Describe the defining characteristics of the projective geometry.

11.6 Define the terms “point at infinity (or ideal point)” and “line at infinity.”

11.7 Describe the properties of parallelism within the meaning of projective geometry.

11.8 Prove theorems regarding projective geometry.

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Unit 12 Topology (OPTIONAL)

General Outcome:

12.0 The student shall be able to (1) describe properties of, and prove theorems about, topological and metric spaces; (2) construct homeomorphisms; and (3) prove theorems regarding topological invariants.

Specific Measurable Learning Outcomes:

Upon successful completion of this unit, the student shall be able to:

12.1 Define the terms “topology” and “topological space.”

12.2 Determine if a space is topological.

12.3 Describe, and prove theorems about, various types of topologies (e.g. the order, product, subspace, and metric topologies).

12.4 Define the terms “metric” & “metric space,” and describe different types of metrics (e.g. the max metric and the taxicab metric).

12.5 Prove theorems regarding topological spaces and metrics.

12.6 Define the term “accumulation point,” and prove theorems regarding accumulation points.

12.7 Define the terms “set,” “subset,” “open subset,” & “closed subset,” and prove theorems regarding open & closed subsets.

12.8 Define the term “limit point,” and prove theorems regarding limit points.

12.9 Define the term “continuous function (or mapping),” and prove theorems regarding continuous functions between topological spaces.

12.10 Define the term “homeomorphism (or topological equivalence),” and prove theorems regarding homeomorphisms.

12.11 Define the term “topological invariant.”

12.12 Define the term “connected space,” and prove theorems regarding connected spaces, including the fact that connectedness is a topological invariant.

12.13 Define the term “compact space,” and prove theorems regarding compact spaces, including the fact that compactness is a topological invariant.